Increasing and Decreasing Functions

Definition. A function is increasing where $f'(x) > 0$ and decreasing where $f'(x) < 0$.

Example

Find the values of x for which $y = x^3 - 6x^2 - 15x$ is decreasing.

Example

$f(x) = x^5 - \frac{3}{x}$, $x \neq 0$. Show that f is increasing for all x in its domain.

Example

Find the range of values of k for which $f(x) = x^3 + kx^2 + 3x$ is increasing for all real x.

Textbook Exercises: SPS Course 6.1, Exercise 4A

Stationary Points

Definition. A stationary point of $y = f(x)$ is a point where $f'(x) = 0$: a maximum, a minimum, or a stationary point of inflection.

Fact (Classifying with the second derivative) — At a stationary point $x = a$:

$f''(a) > 0$	minimum
$f''(a) < 0$	maximum
$f''(a) = 0$	no conclusion — check the gradient either side

Knowledge of the shape of the graph (e.g. a positive cubic) is an acceptable alternative justification.

Example

Find the stationary points of $y = x^3 - 6x^2 + 9x + 2$ and determine their nature. Hence sketch the curve.

Example (Edexcel C2)

The curve with equation $y = x^2 - 32\sqrt{x} + 20,\ \ x > 0$, has a stationary point P. Use calculus to find the coordinates of P, and determine its nature.

Textbook Exercises: SPS Course 6.1, Exercise 4B and Exercise 5 Q1–6

Using Stationary Points

Example

The curve $y = 2x^3 + ax^2 + bx$ has a stationary point at $(1, -4)$.

1. Find a and b.
2. Find the other stationary point and determine the nature of both.

Example

Show that the curve $y = x^4 - 4x^3 + 16x - 3$ has a stationary point at $x = -1$, and determine its nature.

Exercise. $y = x^3$ has $\frac{dy}{dx} = 0$ at the origin, yet the origin is neither a maximum nor a minimum. Check the gradient on each side of $x = 0$, sketch the curve, and explain why the second-derivative test fails here.

Textbook Exercises: SPS Course 6.1, Exercise 5 (remaining) and Exercise 6